

47th International Mathematical Olympiad
Ljubljana, Slovenia

Day I
July 12, 2006

1. Let ABC be a triangle with incentre I . A point P in the interior of the triangle satisfies

$$\angle PBA + \angle PCA = \angle PBC + \angle PCB.$$

Show that $AP \geq AI$, and that equality holds if and only if $P = I$.

2. Let P be a regular 2006-gon. A diagonal of P is called *good* if its endpoints divide the boundary of P into two parts, each composed of an odd number of sides of P . The sides of P are also called *good*.

Suppose P has been dissected into triangles by 2003 diagonals, no two of which have a common point in the interior of P . Find the maximum number of isosceles triangles having two good sides that could appear in such a configuration.

3. Determine the least real number M such that the inequality

$$|ab(a^2 - b^2) + bc(b^2 - c^2) + ca(c^2 - a^2)| \leq M(a^2 + b^2 + c^2)^2$$

holds for all real numbers a , b and c .

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4. Determine all pairs (x, y) of integers such that

$$1 + 2^x + 2^{2x+1} = y^2.$$

5. Let $P(x)$ be a polynomial of degree $n > 1$ with integer coefficients and let k be a positive integer. Consider the polynomial $Q(x) = P(P(\dots P(P(x)) \dots))$, where P occurs k times. Prove that there are at most n integers t such that $Q(t) = t$.
6. Assign to each side b of a convex polygon P the maximum area of a triangle that has b as a side and is contained in P . Show that the sum of the areas assigned to the sides of P is at least twice the area of P .