

**39<sup>th</sup> International Mathematical Olympiad**  
**Taipei, Taiwan**

**Day I**

**July 15, 1998**

1. In the convex quadrilateral  $ABCD$ , the diagonals  $AC$  and  $BD$  are perpendicular and the opposite sides  $AB$  and  $DC$  are not parallel. Suppose that the point  $P$ , where the perpendicular bisectors of  $AB$  and  $DC$  meet, is inside  $ABCD$ . Prove that  $ABCD$  is a cyclic quadrilateral if and only if the triangles  $ABP$  and  $CDP$  have equal areas.
2. In a competition, there are  $a$  contestants and  $b$  judges, where  $b \geq 3$  is an odd integer. Each judge rates each contestant as either “pass” or “fail”. Suppose  $k$  is a number such that, for any two judges, their ratings coincide for at most  $k$  contestants. Prove that  $k/a \geq (b - 1)/(2b)$ .
3. For any positive integer  $n$ , let  $d(n)$  denote the number of positive divisors of  $n$  (including 1 and  $n$  itself). Determine all positive integers  $k$  such that  $d(n^2)/d(n) = k$  for some  $n$ .

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4. Determine all pairs  $(a, b)$  of positive integers such that  $ab^2 + b + 7$  divides  $a^2b + a + b$ .
5. Let  $I$  be the incenter of triangle  $ABC$ . Let the incircle of  $ABC$  touch the sides  $BC$ ,  $CA$ , and  $AB$  at  $K$ ,  $L$ , and  $M$ , respectively. The line through  $B$  parallel to  $MK$  meets the lines  $LM$  and  $LK$  at  $R$  and  $S$ , respectively. Prove that angle  $RIS$  is acute.
6. Consider all functions  $f$  from the set  $N$  of all positive integers into itself satisfying  $f(t^2 f(s)) = s(f(t))^2$  for all  $s$  and  $t$  in  $N$ . Determine the least possible value of  $f(1998)$ .